**Exam. Code : 107201**
**Subject Code : 2067**

## BCA 1st Semester

# APPLIED AND DISCRETE MATHEMATICS

## Paper–III

Time Allowed—3 Hours] [Maximum Marks—75

**Note:–** **Eight** questions are given. Candidates are required to attempt any **five** questions.

1. (a) If A = {1, 2, 3}, B = {4, 5, 6}, C = {7, 8, 9}, then verify that :

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) In a school there are 20 teachers who teach mathematics or physics of these 12 teach mathematics and 4 teach physics and mathematics. How many teach physics ?

(c) Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}. Find $A^C$, $B^C$, $A^C \cap B^C$, $A \cup B$ and hence show that $(A \cup B)^C = A^C \cap B^C$.

(d) If A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8, 10}, C = {1, 2, 3, 4}, then find :

(i) $A - C$

(ii) $A \cap (B - C)$

(iii) $A - (B \cup C)$ 3+4+3+5=15

2. (a) Find $A \Delta B$, if $A = \{2, 3, 5, 7\}$, $B = \{3, 4, 6, 8, 10\}$

(b) Let $A = \left\{\frac{1}{2}, 2\right\}$, $B = \{2, 3, 5\}$, $C = \{-1, -2\}$, then verify that $A \times (B - C) = (A \times B) - (A \times C)$.

(c) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$. Let $R = \{(a, b) : a \in A, b \in B, a \text{ divides } b\}$ be a relation from A into B. Find R. Show that domain of R is A and range of R is B. 5+5+5=15

3. (a) Determine whether the relation represented by zero-one matrix $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ is an equivalence relation.

(b) Let $x = \{1, 2, 3, 4\}$, $R = \{< x, y > \mid x > y\}$. Draw the graph of R and also give its matrix.

(c) Prove that $(p \wedge q) \rightarrow (p \wedge q)$ is a tautology but $(p \vee q) \rightarrow (p \wedge q)$ is not.

(d) Prove the validity of following arguments :

If man is a bachelor, he is unhappy

If a man is unhappy, he dies young

Therefore, bachelors die young 3+4+3+5=15

4. (a) Define two different types of quantifier with example.

(b) Define :

(i) Conjunction

(ii) Disjunction

(iii) Negation

all with truth table.

(c) Write the truth table of following statement :

$[p \rightarrow (q \vee r)] \vee [p \leftrightarrow \sim r]$ 5+5+5=15

5. (a) Prove that $\{[(p \rightarrow q) \vee p] \wedge q\} \rightarrow q$ is a tautology.

(b) Let R be a relation on a set A = {1, 2, 3} defined by :

R = {(1, 1), (1, 2), (2, 3)}. Find the reflexive closure of R and symmetric closure of R.

(c) Define different type of closure with example.

5+5+5=15

6. (a) Show that $(A + B)(\overline{A} + C) = AC + \overline{A}B$

(b) Minimize the function

$f(A, B, C) = \Sigma m(0, 3, 5, 6, 7) + d(2, 4)$

(c) Prove De-morgan law with the help of truth table.

5+5+5=15

7. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that

$$A^3 - 6A^2 + 7A + 2I = 0$$

(b) Given that $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$.

Find AB. Use this to solve the following system of linear equations :

$x - y + z = 4$, $x - 2y = 9$, $2x + y + 3z = 1$

7.5+7.5=15

8. (a) Solve the following system of linear equations by matrix method :

$x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$

(b) Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

7.5+7.5=15